Dialectical proofs accounting for strength of attacks in Argumentation Systems

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Abstract—We consider argumentation systems taking into account several attack relations of different strength. We focus on the impact of various strength attacks on the semantics of such systems, and particularly on the decision problem of credulous acceptance: namely, focussing on one particular argument, a classical issue is to compute a proof, under the form of an admissible set containing this argument. Taking into account attacks of various strength leads to search for the “best” proofs.

Index Terms—Knowledge representation and reasoning, AI algorithms, Argumentation, Dialectical proofs

I. INTRODUCTION

Argumentation has become an influential approach in Artificial Intelligence to model cognitive tasks such as inconsistency handling and defeasible reasoning (e.g. [1], [2], [3]), decision making (e.g. [4]), or negotiation between agents (e.g. [5]).

Argumentation is based on the evaluation of interacting arguments, which may support opinions, claims or decisions. Usually, the interaction takes the form of conflicts between arguments, and the fundamental issue is the selection of acceptable sets of arguments, based on the way they interact. Most of the argumentation-based proposals are instantiations of the abstract system proposed by Dung [6], which is reduced to a set of arguments (completely abstract entities) and a binary relation, called attack, which captures the conflicts between arguments. The increasing interest for the argumentation formalism has led to numerous extensions of the basic abstract system which are more appropriate to the applications.

A first extension of Dung’s system has included a preference relation between arguments, which models their relative strength. For instance, an argument built from certain knowledge is stronger than an argument relying upon default knowledge (see e.g. [7], [8], [9], [10]). Another kind of extension is necessary to make a distinction between various types of conflict. For instance, when arguments are built from logical rules and knowledge, rebut and undercut conflict have been distinguished (see [11], [12]). More generally, symmetric attacks may be considered as weaker than non symmetric attacks, in the sense that non symmetric attacks facilitate the selection of acceptable arguments. [13] has distinguished between blocking attacks and proper attacks, as a consequence of preference between arguments. In a multi-agent setting, various attack relations over a common set of arguments represent different criteria and different contexts according to which the conflicts are perceived (see [14]). Moreover, it is natural to consider that not all attacks are equal in strength. [15] has first suggested the use of weights both on arguments and on attacks. [16] has proposed weighted argument systems, in which attacks are associated with a numeric weight, indicating how reluctant one would be to disregard the attack. Behind these proposals, there is a common idea that attacks may have different strength and can be compared according to their relative strength. However, there is so far no consensus about how it should be used to define extensional semantics, according to which acceptable sets of arguments are selected. A first promising work towards that direction has been proposed in [17], where an abstract argumentation system with varied-strength attacks has been defined. In that novel system, the classical concepts of defence and admissibility are revisited, in different directions, leading to several different refinements.

Our work takes place in that abstract system with attacks of various strength. Focussing on one restriction of the notion of defence, our motivation is to define extensional semantics accounting for the strength of defence, and to study the related decision problem of credulous acceptance. This problem, which consists in deciding if a given argument belongs to a preferred extension of the argumentation system, has been extensively studied within the framework of dialectical proofs (see, for instance, [18], [19], [20] and more recently [21], [22]). In this paper, we investigate the credulous acceptance problem in argumentation systems with attacks of various strength and propose associated dialectical proofs.

In Sect. II, we present the fundamental notions. First, we propose a restricted notion of defence, by requiring that the counter-attack is not weaker than the attack. Then, following Dung’s construction, we define a restricted admissibility, called vs-admissibility, which is used for revisiting the classical preferred semantics and the associated decision problem of credulous acceptance. The next step is to compare defences collectively offered by sets of arguments. So, in Sect. III,
we propose a definition for sets offering a “best” defence for a given argument. Sect. IV is devoted to the description of dialectical proof theories for credulous acceptance under vs-admissibility. A basic one is given which produces a solution under the form of a vs-admissible set containing the queried argument. Then, improvements are proposed in order to produce optimized solutions according to different criteria (set-inclusion minimality and “best” defence for the queried argument). Sect. V is devoted to concluding remarks and some related works. Proofs are omitted for lack of space and can be found in [24].

II. FUNDAMENTALS

We consider the abstract system defined in [17]:

Def. 1 (Argumentation system with attacks of various strength – AS-vs) An argumentation system with attacks of various strength is a triple \( \langle A, \text{ATT}, \geq \rangle \) where \( A \) is a finite set of arguments, \( \text{ATT} \) is a finite set of attack relations \( \langle \rightarrow_1, \ldots, \rightarrow_n \rangle \) on \( A \) and \( \geq \) is a binary relation on \( \text{ATT} \).

Each \( \rightarrow_i \subseteq A \times A \) represents a conflict relation, and \( \geq \) represents a relative strength between these conflict relations. The relation \( \geq \) is only assumed reflexive (it may be partial, and transitive or not). The corresponding strict relation \(^3\) will be denoted by \( \succ \). If the relation \( \geq \) is empty, a classical system (in Dung’s sense) is recovered with the single attack relation obtained as the union of the attack relations \( \rightarrow_i \). In the following of this paper, AS will denote the classical system \( \langle A, R = \bigcup_i \rightarrow_i \rangle \) associated with the argumentation system with attacks of various strength \( \text{AS-vs} = \langle A, \text{ATT}, \geq \rangle \).

Ex. 1 Consider the AS-vs defined by \( A = \{ A, B, C_1, C_2 \} \), \( \text{ATT} = \{ \rightarrow, \rightarrow_1, \rightarrow_2 \} \) with \( \rightarrow_1 = \{(B, A)\} \), \( \rightarrow_2 = \{(C_1, B)\} \), \( \rightarrow = \{(C_2, B)\} \), and \( \geq \) defined by \( \rightarrow \geq \rightarrow_1 \).

\( \text{AS-vs} \) can be depicted by the graph:

An intuitive counterpart to this system can be provided by the following dialogue between the prosecutor and the counsel for the defence during a criminal trial:

Arg. A (Prosecutor): Tom is a suspect since Bob has seen Tom leaving the scene of crime.

Arg. B (Counsel): Bob is myopic, and was too far from the scene of crime; so, he couldn’t see Tom.

Arg. C_1 (Prosecutor’s witness number 1, Bob’s ophthalmologist): Bob has achieved very good results for eye tests; he is not myopic.

Arg. C_2 (Prosecutor’s witness number 2, Bob’s girl friend): Bob does not wear glasses, so he is not myopic.

As regards Bob’s shortsightedness, the opinion of Bob’s ophthalmologist is more reliable than the opinion of Bob’s girl friend. That may lead to state that \( \rightarrow_1 \succ \rightarrow_2 \).

\(^2\) We assume that \( A \) represents the set of arguments proposed by rational agents at a given time; so it makes sense to assume that \( A \) is finite.

\(^3\) \( A \succ B \) iff \( (A \geq B \text{ and not } (B \geq A)) \). \( \succ \) is irreflexive and asymmetric.

In order to convince of the requirement for attacks of various strength, let us consider another example with arguments based on classical logic (see for instance [12]). Each argument is a pair \( (\Phi, \alpha) \) where \( \Phi \) is a consistent set of formulae (called the support) that entails \( \alpha \) (called the claim). One type of attack is the undercut attack. An argument \( A_1 \) undercut an argument \( A_2 \) when the claim of \( A_1 \) entails the negation of the support of \( A_2 \). In [12], the authors have defined measures of the degree of undercut. A degree of undercut is based on a pairwise measure of inconsistency between the claim of the attacker and the support of the attacked argument. This degree of undercut is a notion associated with a pair of arguments. So, this is an example of a weight on an attack relation. For instance, let \( A_1 = \{a, b, (a \land b) \to c\} \), \( A_2 = \{\lnot a\}, \lnot a\), \( A_3 = \{\lnot a, \lnot b\}, \lnot a \land \lnot b\). \( A_2 \) and \( A_3 \) undercut \( A_1 \), but the degree of undercut of the pair \( (A_1, A_1) \) is greater than the degree of undercut of the pair \( (A_2, A_3) \).

Moreover, preferences between attacks cannot always be reduced to preferences between arguments, as shown in the following example:

Ex. 2 Consider the following argumentation system:

\[ A \leftarrow C \rightarrow B \]

In this system, if preferences on attacks come from preferences on arguments, then, assuming that the argument \( C \) is stronger than the argument \( D \), we can consider that the attack from \( C \) to \( A \) is stronger than the attack from \( D \) to \( A \). However, in this case, it is impossible to consider that the attack from \( D \) to \( B \) is stronger than the attack from \( C \) to \( B \). By contrast, this is allowed by argumentation systems with attacks of various strength.

However, in order to handle concrete examples, the relationship between the strength of arguments and the strength of attacks has to be investigated in future works. For instance, it seems rational that a weak argument cannot carry a strong attack.

In this paper, we focus on the strength of attacks. Our purpose is to study the impact of these attacks of various strength on the notion of defence, which is a key concept in argumentation. The first level to be considered is the individual defence level. In Dung’s systems, an argument \( A \) attacked by an argument \( B \) is said defended (against \( B \)) as soon as there exists an argument \( C \) attacking \( B \). Indeed, any attack on \( B \) is relevant for inhibiting the attack from \( B \) to \( A \). Now, if attacks may have different strengths, it is natural to compare the attack on \( B \) with the attack from \( B \) to \( A \). The idea is that some attacks on \( B \) will be too weak to inhibit the attack on \( A \) and thus will not be relevant for reinstating \( A \). Let \( \text{AS-vs} = \langle A, \text{ATT}, \geq \rangle \), the following definition captures the idea of relevant defender:
Def. 2 (vs-defence – vs means “various-strength”) Let $A$, $B$, $C \in A$ such that $C \nrightarrow B$ and $B \nrightarrow A$. $C$ vs-defends $A$ against $B$ (or $C$ is a vs-defender of $A$ against $B$) iff $\forall j \nrightarrow i \rightarrow j$ (i.e. the attack from $B$ to $A$ is not strictly better than the one from $C$ to $B$).

Note that the same kind of definition is encountered in works about preference-based argumentation, for combining attack relation and preference relation (see [7], [9]): in this context, the idea is that an attack from $B$ to $A$ is relevant if $A$ is not strictly preferred to $B$; otherwise, it can be considered that $A$ defends itself against $B$.

So, in a similar way, Def. 2 states that the attack from $B$ to $A$ is overruled by the attack from $C$ to $B$ if the attack from $B$ to $A$ is not strictly better than the attack from $C$ to $B$.

Note that Def. 2 can be restated in the system proposed by [17], where defenders are classified in four categories: strong, weak, normal and unqualified. Our notion of vs-defender exactly corresponds to “not weak defender”, or equivalently to “strong or normal or unqualified defender”.

As required above, the notion of vs-defence refines the classical notion of defence.

Prop. 1 Let $A$, $B$, $C \in A$. If $C$ vs-defends $A$ against $B$ then $C$ defends $A$ against $B$ in Dung’s sense.

Then, following the classical construction of acceptability (or collective defence), we propose to refine Dung’s classical notions of acceptability and admissibility.

Def. 3 (vs-acceptability and vs-admissibility) Let $S \subseteq A$ and $A \in A$.

- $A$ is vs-acceptable with regard to (wrt) $S$ (or $S$ collectively vs-defends $A$ against any attack) iff $\forall B \in A$, if $B$ attacks $A$ then $\exists C \in S$ such that $C$ vs-defends $A$ against $B$.
- $S$ is vs-admissible iff $S$ is conflict-free in $AS^v$ and $\forall A \in S$, $A$ is vs-acceptable wrt $S$.

The notion of vs-acceptability is a particular case of the notion of “constrained acceptability” of [17]. Indeed, it is sufficient to consider the defence profile containing the three levels strong, normal and unqualified. Moreover, the vs-admissibility requires the classical strict notion of conflict-free. That ensures that for any vs-admissible set $S$, no attack may occur between elements of $S$. Note also that vs-acceptability and vs-admissibility refine the corresponding classical notions (this is a direct consequence of Prop. 1):

Prop. 2 If $S \subseteq A$ is vs-admissible, then $S$ is admissible in Dung’s sense.

The converse is false: Let $C \nrightarrow B \nrightarrow A$, with $\forall j \nrightarrow i \rightarrow j$; $\{C, A\}$ is admissible but not vs-admissible. Note also that the empty set is vs-admissible.

Admissibility is the basis of most of the classical semantics, in Dung’s abstract system. So, it is straightforward to revisit some classical semantics, using the notion of vs-admissibility. For instance, let us consider the preferred semantics which produce maximal (for set-inclusion) admissible sets of arguments.

Def. 4 (preferred vs-extension) Let $S \subseteq A$ be a vs-admissible set. $S$ is a preferred vs-extension of $AS^v$ iff $\exists S' \subseteq A$ such that $S \subseteq S'$ and $S'$ is vs-admissible.

Ex. 1 (cont) With $\nrightarrow \nrightarrow i \rightarrow j$ and $k \rightarrow j \rightarrow k$, $C_1$ and $C_2$ are vs-defenders of $A$. With $\nrightarrow \nrightarrow i \rightarrow k \rightarrow k$, $C_1$ is the only vs-defender of $A$. In both cases, the set $\{A, C_1, C_2\}$ is the only preferred vs-extension.

Preferred vs-extensions have nice properties:

Prop. 3 Preferred vs-extensions satisfy the following points:

1) Let $S \subseteq A$ be a vs-admissible set, there exists a preferred vs-extension $E$ of $AS^v$ such that $S \subseteq E$.
2) There always exists at least one preferred vs-extension of $AS^v$.
3) For each preferred vs-extension $E$ of $AS^v$, there exists a preferred extension $E'$ of $AS$ such that $E \subseteq E'$.

At this point, it is important to say that there does not exist a formal compilation of an $AS^v$ into a classical Dung’s argumentation framework $AR = (A, \rightarrow)$ (over the same arguments) which would be equivalent, i.e. such that the vs-admissible sets of $AS^v$ would coincide with the admissible sets of $\{A, \rightarrow\}$. The following example illustrates the impossibility to obtain such a translation:

Ex. 3 Consider the $AS^v$ represented by

\[
\begin{align*}
C_1 & \xrightarrow{j} B & i & \xrightarrow{j} A_1 \\
C_2 & \xrightarrow{h} B & k & \xrightarrow{i} A_2
\end{align*}
\]

with:

- $\nrightarrow \nrightarrow i \rightarrow j$ ($C_1$ is not a vs-defender of $A_1$),
- $\nrightarrow \nrightarrow k \rightarrow i$ ($C_1$ is a vs-defender of $A_2$),
- $\nrightarrow \nrightarrow h \rightarrow i$ ($C_2$ is a vs-defender of $A_1$),
- $\nrightarrow \nrightarrow h \rightarrow k$ ($C_2$ is a vs-defender of $A_2$).

Assuming that $AS^v$ can be compiled into an equivalent classical system $AR = (A, \rightarrow)$, the following constraints hold:

- $\{C_1, C_2, A_1, A_2\}$ is vs-admissible, so $\{C_1, C_2, A_1, A_2\}$ must be admissible in $AR$, and so $\{C_1, C_2, A_1, A_2\}$ must be conflict-free in $AR$.
- $\{A_2\}$ is not vs-admissible, so $\{A_2\}$ should not be admissible in $AR$. So, $A_2$ must be attacked and since $\{C_1, C_2, A_1, A_2\}$ is conflict-free, $A_2$ is attacked by $B$ in $AR$, and $B$ is the only attacker of $A_2$ in $AR$.
- And similarly, $B$ is the only attacker of $A_1$ in $AR$. 

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4In Dung’s work [6]: a set $S \subseteq A$ is conflict-free if $\forall A, B \in S$, $B$ does not attack $A$; $S$ is admissible if it is conflict-free and defends all its elements against all the possible attacks.

5I.e., $\forall A, B \in S, \exists \nrightarrow \in \text{ATT}, \text{s.t. } B \nrightarrow A$. 

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\{C_1, A_2\} is vs-admissible, so \{C_1, A_2\} must be admissible in AR. So, C_1 must attack B in AR.

\{C_1, A_1\} is not a vs-admissible set, so \{C_1, A_1\} should not be admissible in AR.

As C_1 attacks B, which is the only attacker of A_1, and due to the fact that \{C_1, C_2, A_1, A_2\} is conflict-free, it must be the case that C_1 does not defend C_1. The only possibility is that C_1 is attacked by B (since \{C_1, C_2, A_1, A_2\} is conflict-free). But then, the set \{C_1, A_1\} would still be admissible (since C_1 attacks B).

So, in conclusion, it is not possible to translate the above AS_v to an equivalent classical system.

Finally, we consider the well-known decision problem called credulous acceptance problem [20], that is “deciding if a given argument belongs to (at least) one preferred extension”. So, under vs-admissibility, it comes to decide if a given argument belongs to (at least) one preferred vs-extension, or equivalently, due to Prop. 3:

**Def. 5 (Credulous vs-acceptability)** Under vs-admissibility, an argument is credulously accepted if it belongs to (at least) one vs-admissible set.

In order to be able to solve this credulous vs-acceptance problem, we define in Sect. IV proof theories, inspired by the work of [19]. We want to propose several kinds of proof criteria to satisfy. Note that Def. 8 supports a priori neither the minimal sets for set-inclusion, nor the maximal sets for set-inclusion. Our aim is to use the scheme of Def. 8 for comparing sets of vs-defenders of a given argument A. So, we instantiate this scheme by replacing the relation is strictly better than by the strict relation given in Def. 6. Thus, S_1 and S_2 being two sets which collectively vs-defend A, we are able to determine whether S_1 offers a better defence for A than S_2, using the comparison of Def(\(A, S_1\)) and Def(\(A, S_2\)) by the appropriate instantiation of Def. 8:

**Def. 9 (Set-comparison wrt the defence of an argument)** Let \(S_1 \subseteq A\) and \(S_2 \subseteq A\) such that \(A\) is vs-acceptable wrt \(S_1\) and \(S_2\). S_2 strictly better than \(S_1\) wrt the defence of \(A\) will be denoted by \(S_2 \gg A S_1\), and is defined by:

\[
S_2 \gg A S_1 \iff \text{Def}(A, S_2) \supseteq \text{Def}(A, S_1)
\]

Note that, due to the definition of \(\supseteq\), the relation \(\gg A\) is irreflexive and asymmetric.

Given an argument \(A\) and using Def. 9, we are able to compare two vs-admissible sets which defend \(A\). We will take advantage of this comparison in the search for a proof
that $A$ is credulously vs-accepted. Such a proof could simply be a vs-admissible set containing $A$. A proof exhibiting a best defence for $A$ would be more informative. Moreover, from a computational point of view, shorter proofs are more satisfactory, since searching for defenders is expensive. Combining the two requirements will lead us to search for proofs proposing a minimal set of defenders and a best defence for the queried argument. This discussion is illustrated on the following example:

**Ex. 4** This example has been taken from [17].

\[
\begin{array}{c}
\xymatrix{ & C \ar[dl]_j \ar[dr]^k & B \ar[dl]_i \ar[dr]^j & F \ar[dl]_i \ar[dr]^k & A \ar[d]_j \ar[l]^m \ar[r]_n \ar[r]_p & D \ar[l]_j \ar[r]_i \ar[r]_k & A & F. }
\end{array}
\]

Assume that $C$ and $D$ are two vs-defenders of $A$ and $F$.

{$\{A, C, F\}$ and $\{A, D, F\}$ are preferred vs-extensions containing $A$, so they are proofs for the credulous vs-acceptance of $A$. However, from a computational point of view, more interesting vs-admissible sets are $\{A, C\}$ and $\{A, D\}$ for proving that $A$ is credulously vs-accepted. Now, assuming that $\frac{j}{i} \succ k$, we have $\{A, C\} \succ_A \{A, D\}$ and the best proof is $\{A, C\}$ (since $C$ is a better vs-defender of $A$ than $D$).

In the following, we formalize the above ideas, and first of all the concept of minimal vs-admissible set containing a given argument. As far as we know, very few works have addressed the credulous acceptance problem by the computation of minimal lines of defence. [27] has proposed an algorithm for computing minimally admissible defence sets, using the concept of a defence set around $A$. This is exactly what we call an $A$-min-admissible set (an admissible set which contains $A$ and which is $\subseteq$-minimal among the admissible sets containing $A$). Then, taking into account vs-defence leads to:

**Def. 10** ($A$-min-vs-admissible set) Let $AS_{vs}$ be an argumentation system with attacks of various strength. Let $A \in A$ and $S \subseteq A$. $S$ is $A$-min-vs-admissible iff

1) $S$ is vs-admissible,
2) $A \in S$
3) $S$ is $\subseteq$-minimal among the sets satisfying the two previous conditions.

Then, using Def. 9, we are able to compare vs-admissible sets which minimally defend $A$. The sets which are maximal for the relation $\succ_A$ among the min-vs-admissible sets, called $A$-min-best-defences, correspond to the best proofs we are looking for.

**Def. 11** ($A$-min-best-defence) $S \subseteq A$ is a $A$-min-best-defence iff $S$ is $A$-min-vs-admissible and $\nexists S' \subseteq A$ $A$-min-vs-admissible, such that $S' \succ_A S$ (i.e. such that Def$(A, S') \supseteq$ Def$(A, S)$)

The above definitions are illustrated on the following examples.

In Ex. 1, if $C_2$ is not a vs-defender of $A$, then $\{C_2, A\}$ is $A$-min-admissible but not $A$-min-vs-admissible. If $C_1$ and $C_2$ are both vs-defenders of $A$, and $C_1$ is strictly better than $C_2$, then $\{C_1, A\}$ is the only $A$-min-best-defence, whereas, if $C_1$ and $C_2$ are equivalent or uncomparable, $\{C_1, A\}$ and $\{C_2, A\}$ are both $A$-min-best-defences.

In Ex. 4, $\{A, C\}$ and $\{A, D\}$ (resp. $\{F, C\}$ and $\{F, D\}$) are both $A$-min-vs-admissible (resp. $F$-min-vs-admissible). If $\frac{j}{i}$ and $\frac{k}{l}$ are equivalent or uncomparable, $\{A, C\}$ and $\{A, D\}$ are both $A$-min-best-defences. But, if $\frac{j}{i} \succ k$, $\{A, C\}$ is the only $A$-min-best-defence (and $\{F, D\}$ is the only $F$-min-best-defence). Note that, in that case, the $A$-min-vs-admissible set $\{A, D\}$ is not included in an $A$-min-best-defence.

Note that an $A$-min-best-defence does not always contain the best vs-defenders of $A$, since vs-admissibility is emphasized first. As shown on the following example, the best vs-defenders of $A$ may not belong to a vs-admissible set.

**Ex. 5** Assume that all the $C_i$ are vs-defenders of $A$, and $\frac{j}{i} \succ k_i$. $\{A, C_2\}$ is the only $A$-min-vs-admissible set and so it is also the only $A$-min-best-defence. And yet $C_2$ is not the best defender of $A$ against $B$. However, choosing $C_1$ would lead to a set which is not vs-admissible.

A variant of this example also illustrates an important point: the minimality in Def. 10 is the minimality wrt the set-inclusion and not wrt the cardinality: it is sufficient to remove the argument $D$ of the previous argumentation system for obtaining two $A$-min-vs-admissible sets, $\{A, C_1, C_3\}$ and $\{A, C_2\}$, and only one $A$-min-best-defence, $\{A, C_1, C_3\}$.

The last remark concerns the existence of the $A$-min-best-defences, which is not guaranteed:

**Ex. 6**

\[
\begin{array}{c}
\xymatrix{ & C_1 \ar[d]_j \ar[r]^k & C_2 \ar[d]_l \ar[r]^j & C_3 \ar[d]_l \ar[r]^k & B \ar[d]_l \ar[r]^j & A \ar[d]_l \ar[r]^k & A }
\end{array}
\]

Assume that $\frac{1}{i} \succ \frac{j}{k}, \frac{k}{l} \succ \frac{j}{i}$, and $\frac{j}{i} \succ \frac{k}{l}$. $\{A, C_1\}$, $\{A, C_2\}$, $\{A, C_3\}$ are the three $A$-min-vs-admissible sets and one has the following preference between these sets: $\{A, C_1\} \succ_A \{A, C_2\} \succ_A \{A, C_3\} \succ_A \{A, C_1\}$. So there is no maximal set for $\succ_A$ and, so no $A$-min-best-defence.

**IV. Dialectical proofs**

**A. Dialectical framework**

A proof of credulous acceptance for an argument $A$ can be presented under a dialectical form (see for instance [18], [20], [19]) where two players exchange arguments. The proponent (PRO) proposes the initial argument $A$ and the arguments which directly or indirectly defend $A$ in the proof. The

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7Note that this existence is not guaranteed even if the relation $\succeq$ between the attack relations is transitive (see the example given in the appendix of [24]).
A dialogue type gives a formal framework for the definition of rules, using the legal-move function. Then, it is necessary to define formal conditions under which a given \( \phi \)-dialogue is successful. These conditions are usually called winning criteria. We consider the first criterion given in [18], [19].

**Def. 13 (winning criterion)** Let \((A, ATT, \succeq, \phi)\) be a dialogue type. A \( \phi \)-dialogue \( d \) is won by PRO if and only if \( d \) is finite, cannot be continued (i.e. \( \phi(d) = \emptyset \)), and \( \text{Pl}(\text{Last}(d)) = \text{PRO} \).

Combining a dialogue type \((A, ATT, \succeq, \phi)\) and a winning criterion, we obtain a \( \phi \)-dialectical proof theory and \( \phi \)-proofs. So, according to Def. 13, we define:

**Def. 14 (\( \phi \)-proof)** A \( \phi \)-proof is a \( \phi \)-dialogue won by PRO.

**B. Proofs of credulous \( \phi \)-acceptance: from “basic proofs” to “best proofs”**

The credulous vs-acceptance problem comes to “decide if a given argument \( X \) belongs to (at least) one vs-admissible set”. This problem can be solved following different methods, the first two ones simply correspond to dialectical proofs and the third one is the composition of a dialectical proof and a filtering process:

- The first method consists in finding a proof that \( X \) is credulously vs-accepted and it is sufficient that this proof exhibits a vs-admissible set containing \( X \). Thus, we can start from the dialectical proof theory proposed in [19] and add constraints to the legal-move function, in order to ensure that PRO always advances vs-defenders of arguments she previously advanced. Thus, we can define \( \phi^{\text{\( \pi \)}} \)-proofs for credulous vs-acceptance (called “basic proofs”).

- The second method is the production of proofs for credulous vs-acceptance which are “less expensive” than \( \phi^{\text{\( \pi \)}} \)-proofs; in this case, we want to exhibit a X-min-vs-admissible set, so a more constrained legal-move function is proposed corresponding to \( \phi^{\text{\( \pi \)}} \)-proofs (called “minimal proofs” because they must respect an additional criterion: the set-inclusion minimality).

- The last method should take into account another criterion: the quality of the defence, in the sense that this method should concern the production of “best proofs” (i.e. proofs exhibiting a X-min-best-defence). However, for that purpose, the dialectical framework defined above is not appropriate. Indeed, there is no simple way to obtain such best proofs by adding new constraints on the legal-move function, as shown by Ex. 5, where PRO does not advance the best vs-defender of \( A \) when she builds the best proof for \( A \). So, in this paper, we propose to determine best proofs for a given argument \( X \) by comparing the \( \phi^{\text{\( \pi \)}} \)-proofs for \( X \).

Before the basic definition for \( \phi^{\text{\( \pi \)}} \)-legal move functions (\( i = 1, 2 \)), let us give some useful notations.

**Notation 1** Let \( d = \mu_0 \mu_1 \ldots \mu_i \) be a finite \( \phi \)-dialogue:

- \( \mu_i \) is denoted by \( \text{Last}(d) \);
- \( \phi(\text{Att}(\mu_0) \ldots \text{Att}(\mu_i)) \) is denoted by \( \phi(d) \);
- \( \text{PRO}(d) \) denotes the set of arguments advanced by PRO during \( d \);
- The extension of \( d \) with a move \( \mu \) in \( AS_{\text{vs}} \) such that \( \mu_0 \ldots \mu_i \mu \) is a \( \phi \)-dialogue is denoted by the juxtaposition \( d, \mu \).
Notation 2 Let AS denote the classical system \((A, R = \bigcup_i \rightarrow_i \}) associated with the argumentation system with attacks of various strengths \(AS_{vs} = \langle A, AT\T, \succeq \rangle\).

- Let \(A \in A\), \(R^+(A)\) (resp. \(R^-(A)\)) denotes the set of arguments which are attacked by \(A\) (resp. which attack \(A\)) in the sense of \(R\); \(R^\pm(A) = R^+(A) \cup R^-(A)\);
- Let \(S \subseteq A\), \(R^+(S)\) denotes the set of arguments which are attacked by an element of \(S\); \(R^-(S)\) and \(R^\pm(S)\) are defined in an analogous way;
- \(\text{Ref}\) denotes the set of arguments which are self-attacking;
- \(\text{vsDef}(A, B)\) will denote the set of the vs-defenders of \(A\) against \(B\).

Following [19], we state the following constraints which must be verify for each \(\phi_i^{vs}\)-dialogue. Every move \([\text{OPP}, (B, A)]\) must reply to a preceding move \([\text{PRO}, (A, F)]\) in the dialogue, that is \(\text{OPP}\) advances \(B\) which attacks an argument \(A\) previously advanced by \(\text{PRO}\). Every move \([\text{PRO}, (C, B)]\), except the first one, must be immediately preceded in the dialogue by a move \([\text{OPP}, (B, A)]\) such that \(C\) is a vs-defender of \(A\) against \(B\).

Moreover, \(\text{PRO}(d)\) must be vs-admissible, so \(\text{PRO}\) cannot choose any argument in \(R^\pm(\text{PRO}(d))\), nor any self-attacking argument. And it is useless for \(\text{PRO}\) to advance an argument previously advanced by \(\text{PRO}\). It is also useless for \(\text{OPP}\) to play an attack \((B, A)\) if \(\text{PRO}(d)\) contains a vs-defender of \(A\) against \(B\).

In order to produce “minimal proofs” for the credulous vs-acceptance problem, the idea is to enforce the restriction on moves by \(\text{PRO}\) as follows: if \(\text{PRO}\) plays an attack \((C, B)\) with \(C\) authorized by \(\phi_i^{vs}\), \(\text{PRO}\) will not be permitted to advance any argument from \(\text{vsDef}(A, B)\) in the continuation of the dialogue.

So, we propose the following definition for legal-move functions:

**Def. 15** \((\phi_i^{vs}\)-dialogues \((i = 1, 2)\) \(\phi_i^{vs} : (A \times A)^* \rightarrow 2^{(A \times A)}\) are defined by:

- if \(d\) is a dialogue about the argument \(X\) of odd length (next move is by \(\text{OPP}\)), \(\phi_i^{vs}(d) = \{(B, A) \in A \times A \mid A \in \text{PRO}(d)\) and \(B \in R^-(A)\) such that \((\text{vsDef}(A, B) \cap \text{PRO}(d))\) is empty};
- if \(d\) is a dialogue about the argument \(X\) of even length (next move is by \(\text{PRO}\)) with \(\text{Att}(\text{Last}(d)) = (B, A)\), \(\phi_i^{vs}(d) = \{(C, B) \in A \times A \mid C \in \text{vsDef}(A, B) \cap \text{POSS}(d)\},\) with
  - \(\text{POSS}_1(d) = A \setminus \{\text{Ref} \cup \text{PRO}(d) \cup R^\pm(\text{PRO}(d))\}\)
  - \(\text{POSS}_2(d) = \text{POSS}_1(d) \setminus \{X \in \text{vsDef}(Y, Z) \mid \text{vsDef}(Y, Z) \cap \text{PRO}(d) \neq \emptyset\}\)

Note that each \(\phi_2^{vs}\)-dialogue is also a \(\phi_1^{vs}\)-dialogue. Moreover, the restriction on moves by \(\text{OPP}\) is the same in the two dialogue types.

The following results enable us to establish the soundness and the completeness of these two \(\phi_i^{vs}\)-proof theories (the proofs of these results are given in the appendix of [24]).

**Lem. 1** If \(d\) is a \(\phi_1^{vs}\)-dialogue, \(\text{PRO}(d)\) is conflict-free.

**Theo. 1** (Soundness of \(\phi_i^{vs}\)-proofs) Let \(X \in A\).

1. If \(d\) is a \(\phi_1^{vs}\)-proof for \(X\), then \(\text{PRO}(d)\) is a vs-admissible set containing \(X\).
2. If \(d\) is a \(\phi_2^{vs}\)-proof for \(X\), then \(\text{PRO}(d)\) is a X-min-vs-admissible set.

**Theo. 2** (Completeness of \(\phi_i^{vs}\)-proofs) Let \(X \in A\).

1. If \(X\) is in a vs-admissible set, then there exists a \(\phi_1^{vs}\)-proof for \(X\).
2. If \(X\) belongs to \(S\), a X-min-vs-admissible set, then there exists a \(\phi_2^{vs}\)-proof \(d\) for \(X\) such that \(\text{PRO}(d) = S\).

The following example illustrates the above theorem by showing the \(\phi_2^{vs}\)-proof which is built.

**Ex. 7** Consider the following argumentation system in which we assume that \(Y_1, Y_2, Y_3\) are vs-defenders of \(X\), and that \(Y_2\) is a vs-defender of \(Y_3\):

\[
\begin{array}{ccc}
Y_1 & \longrightarrow & Z_1 \\
Y_2 & \longrightarrow & Z_2 \\
D & \longrightarrow & Y_3 \\
& \longrightarrow & Z_3 \\
& \longrightarrow & X
\end{array}
\]

Let \(S = \{Y_1, Y_2, Y_3, X\}\). \(S\) is a X-min-vs-admissible set. So, there is a \(\phi_2^{vs}\)-proof \(d\) for \(X\) such that \(\text{PRO}(d) = S\). Here is \(d\):

<table>
<thead>
<tr>
<th>Move (\mu_i)</th>
<th>(\phi_2^{vs}(\mu_1, \ldots, \mu_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_0 = \text{PRO}.(X, \text{___}))</td>
<td>{({Z_1, X}, {Z_2, X}, {Z_3, X})}</td>
</tr>
<tr>
<td>(\mu_1 = \text{OPP}.(Z_1, X))</td>
<td>{({Y_1, Z_1})}</td>
</tr>
<tr>
<td>(\mu_2 = \text{PRO}.(Y_1, Z_1))</td>
<td>{({Z_3, X})} (\text{OPP cannot advance} (Z_2) \text{since} (Y_1 \in \text{vsDef}(X, Z_2)))</td>
</tr>
<tr>
<td>(\mu_3 = \text{OPP}.(Z_3, X))</td>
<td>{({Y_2, Z_3})}</td>
</tr>
<tr>
<td>(\mu_4 = \text{PRO}.(Y_2, Z_3))</td>
<td>{({D, Y_2})}</td>
</tr>
<tr>
<td>(\mu_5 = \text{OPP}.(D, Y_2))</td>
<td>{({Y_2, D})}</td>
</tr>
<tr>
<td>(\mu_6 = \text{PRO}.(Y_2, D))</td>
<td>(\emptyset)</td>
</tr>
</tbody>
</table>

Concerning the production of “best proofs” for \(X\), we propose the following methodology. Due to the above results, we know that if there exists \(S\), a X-min-best-defence, \(S\) is X-min-vs-admissible. So, there exists a \(\phi_2^{vs}\)-proof \(d\) for \(X\) such that \(\text{PRO}(d) = S\). Thus, we just have to compute all the \(\phi_2^{vs}\)-proofs \(d_i\) for \(X\), and to compare the sets \(\text{PRO}(d_i)\) with the relation \(\Rightarrow_X\).

As said above, the dialectical framework proposed in this paper is not appropriate for designing a proof theory able to produce X-min-best-defences, when they exist. The main reason is that \(\text{PRO}\) should not be permitted to advance a vs-defender for which there exists no \(\phi_0^{vs}\)-proof. Indeed, due to Def. 13, a \(\phi\)-proof is a sequential proof. And it is not possible to backtrack on the choice of a vs-defender advanced by \(\text{PRO}\).
We are currently investigating another dialectical framework, using tree-like proofs.

V. CONCLUSION AND RELATED WORKS

Our proposal in this paper is a further contribution to the development of argumentation with various attacks of different strength, based on the abstract framework introduced by [17]. The basic idea is to use the relative strength of the attacks for refining the concept of reinstatement: we define a new notion of defence, the vs-defence, requiring that the counterattack is not weaker than the attack. This enables us to revisit Dung’s classical semantics with the definition of vs-admissible sets, preferred vs-extensions, and the credulous vs-acceptance problem.

Another issue is to compare the defence offered by sets of arguments. We propose comparisons at different levels: between vs-defenders, between sets of vs-defenders and between vs-admissible sets.

This enables us to define two dialectical proof theories for credulous vs-acceptance. The first one improves the $\phi_1$-proof theory proposed in [19] by taking into account the new notion of defence. So, a $\phi_2^{vs}$-proof for a given argument $A$ produces a vs-admissible set containing $A$. Then, we propose the $\phi_2^{vs}$-proof theory, a more constrained proof theory which produces $\subseteq$-minimal sets among the vs-admissible sets containing $A$. For the $\phi_1^{vs}$ and $\phi_2^{vs}$-proofs, theorems of soundness and completeness are given.

The last issue concerns the production of $A$-min-best-defences, which represent the best proofs for $A$. These proofs cannot be easily obtained by a simple restriction of the $\phi_2^{vs}$-dialogues but can be computed by comparing the $\phi_2^{vs}$-proofs for $A$.

To our best knowledge, this is the first attempt to construct best proofs for the credulous acceptance problem. [27] has proposed a procedure for computing minimal lines of defence around a given argument. But this procedure does not use a dialectical method, and does not always produce minimal admissible sets. For many years, there has been substantial research on dialectical proof procedures for classical abstract argumentation. Recently, [21], [22] have proposed a unifying framework able to capture dialectical proof procedures for handling both credulous and skeptical acceptance in abstract argumentation. This framework is based on the notions of dispute derivation and base derivation. Dispute derivation provides a way to define proofs, and is very close to the approach of [19], which has inspired our work. Base derivation enables to represent backtracking in the search for a proof, and is used for proving skeptical acceptance. As a future work, we plan to consider this powerful unifying framework for obtaining proofs accounting for strength of attacks. Another future work will concern both the study of worst-case complexity and the development of algorithms dedicated to the proof theories presented in this paper.

REFERENCES